California Institute of Technology
Transfer Admissions
Sample Exam Questions

THIS IS NOT THE ACTUAL EXAM. These sample questions are provided for transfer applicants to **prepare** for the types of questions that will appear on the actual exam.

On the actual exam we expect to see all work, as we are interested in methods as well as answers.

PLEASE NOTE: The actual exam will be **CLOSED BOOK**: No reference materials, calculators, electronic devices, or other aids are allowed. If an applicant requires special testing accommodations, they must contact the Office of Undergraduate Admissions by the application deadline.

Caltech Transfer Admissions
Mathematics Examination
Sample Questions

Basic Calculus

1) Find the maximum and minimum values of the function

$$f(x) = \frac{\sin^2(x)}{\sqrt{2}} + \cos(x)$$

on the interval $[0, \pi]$. Describe the intervals in which f is increasing or decreasing. Sketch the graph of f.

Linear Algebra

1) The vectors (−1, 0, 1) and (0, −1, 0) are solutions of the linear system

$$\begin{aligned} x - 2y + 3z &= 2 \\ 2x + y + z &= -1 \\ x - y + 2z &= 1 \end{aligned}$$

a) Find *all* solutions of the system

b) What is the solution vector of the shortest length?

c) Find a nonzero vector which is perpendicular to *all* solution vectors of the same system.

Vector Calculus

1) Let C be the helix in $\mathbb{R}^3$ parametrized by $\alpha(t) = \frac{1}{\sqrt{2}}(\cos(t), \sin(t), t)$, $0 \leq t \leq 2\pi$. Find the arc length $s(t)$, the unit tangent vector $T(t)$ and the curvature $k(t)$ (this is defined as $||T'(t)||$). Evaluate $s(\pi)$, $T(\pi)$, $k(\pi)$.

Differential Equations

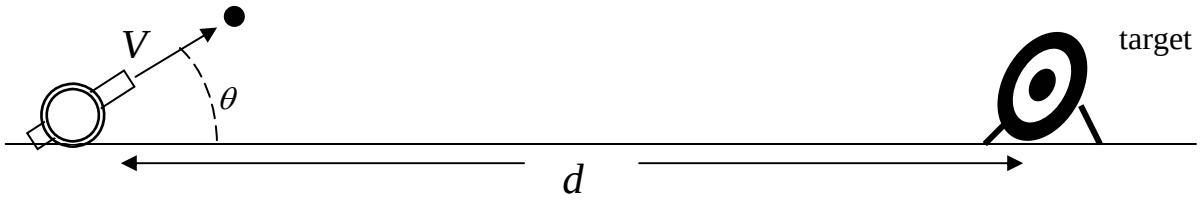
1) For which real values of a do there exist solutions of the differential equation

$$y'' + 2y' + ay = 0$$

which satisfy the conditions $y(0) = y(\pi) = 0$ but which are not identically zero? For each such a give an appropriate non-zero solution

PROBLEM 1

Your job is to fire a cannon and hit a target a distance d away. The cannon's muzzle velocity is V. The cannon and the target are on the same level.



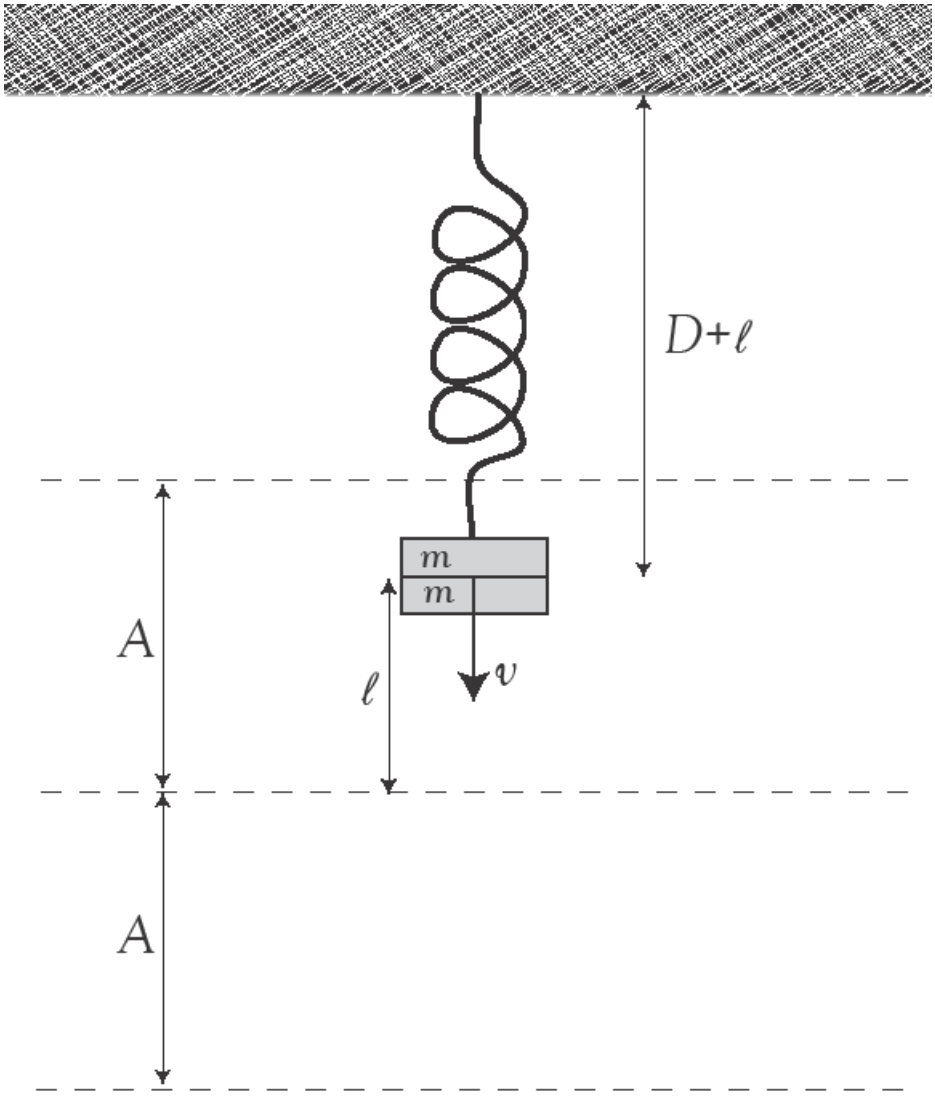
a) For a given angle θ, what is the muzzle velocity $V(\theta)$ that is required to hit the target?

b) What is the minimum muzzle velocity V_0 that is required to hit the target? For what angle θ is it achieved?

c) Suppose $V = \sqrt{2}\, V_0$. Determine the firing angles θ for which success will be achieved.

PROBLEM 2

A linear spring has a length *D*. When a mass *m* is hung on one end, the spring has an equilibrium length *D + l*. While it is hanging motionless with an attached mass *m*, a second mass *m* is dropped from a height *l* onto the first one. The masses collide inelastically and stick together. The figure below shows the system at the time of the collision. Subsequently it oscillates with an amplitude *A* about its new equilibrium position.



PROBLEM 2 (continued)

(a) Explain why the new equilibrium length of the spring is $D + 2l$, as indicated in

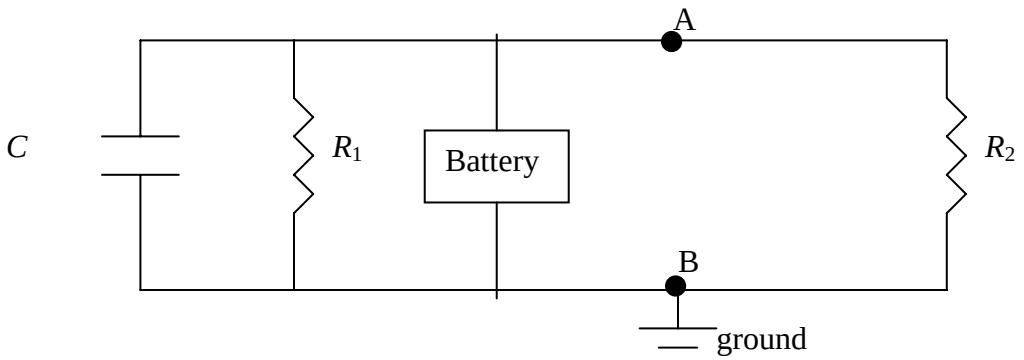
(b) What is the period of the resulting motion?

(c) Find the amplitude A of the motion. Express your answer in terms of l.

(d) How long after the collision do the joined masses reach the lowest point of their oscillation? Express your answer in terms of l and g.

PROBLEM 3

Consider the following electrical circuit, containing a capacitor, two resistors, and a battery of voltage V:



a) In steady state what is the charge on the capacitor and what is the current through each of the resistors?

b) Starting from the steady state operation of part a), the battery is disconnected at time $t = 0$. What is the voltage at point A for all $t > 0$?

c) Now suppose that $C = 100$ pF and $R_1 = R_2 = 10$ MΩ. Calculate the amount of time in which the voltage at A decreases by a factor of e.